Key Stage 2 - Written Calculation Policy

(Aligned with the 2014 National Curriculum)
<Year 3 Progression from ‘Inspire Maths’ in KS1>
<Year 4 onwards progression from National Curriculum in KS1>

Written Methods adapted from a policy written by Diane Andrews, Maths Consultant
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Bar Method Methods adapted from the Oxfordshire Maths Teams

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Policy Review: JULY 2019
Addition - Year Three

Number Line Method

Introduce empty number line strategy.

78 + 34 = 112

Further develop with addition of a three-digit and a two-digit number:

125 + 56 = 181

Formal Written Method (2 digits + 2 digits, 3 digits + 3 digits)

Continue to build on understanding from ‘Inspire Maths’ using Formal Written Method.

1) No carrying required (TU+TU)
2) Carrying required (TU+TU)
3) Carrying required (HTU+HTU)
The digits that have been ‘carried’ should be recorded under the line in the correct column.

1) \[ 27 + 12 = \]
   \[
   \begin{array}{c}
   27 \\
   +12 \\
   \hline
   39
   \end{array}
   \]

2) \[ 44 + 76 = \]
   \[
   \begin{array}{c}
   44 \\
   +76 \\
   \hline
   120
   \end{array}
   \]

3) \[ 476 + 147 = 623 \]
   \[
   \begin{array}{c}
   447 \\
   +176 \\
   \hline
   623
   \end{array}
   \]

‘Seven add six equals 13. Write three in the units column and ‘carry’ one across into the tens column (10).

40 add 70 and the ten that we carried equals 120. Write 2 in the tens column (20) and ‘carry’ 1 across into the hundreds column (100).

400 add 100 and the 100 that has been carried equals 600. Write 6 in the hundreds column (600). The total is 623.
Addition - Year Four

Number Line Method

Introduce empty number line strategy (see Year 3).

125 + 56 = 181

\[\begin{array}{c}
125 \\
+50 \\
175 \\
+6 \\
181
\end{array}\]

1,845 + 526 = 2,371

\[\begin{array}{c}
1845 \\
+500 \\
2345 \\
+20 \\
2365 \\
+6 \\
2371
\end{array}\]

Partitioning (building towards the Formal Written Method)

Initially use calculations where it is not necessary to bridge across the tens or hundreds:

63 + 32 = 95

\[\begin{array}{c}
60 + 3 \\
+30 + 2 \\
90 + 5 = 95
\end{array}\]

\[\begin{array}{c}
63 \\
+32 \\
95
\end{array}\]

\[\begin{array}{c}
6 \quad 3 \\
+ \quad 3 \quad 2 \\
5 \quad (3 + 2) \\
9 \quad 0 \quad (60 + 30) \\
9 \quad 5
\end{array}\]
Formal Written Method (2 digits + 2 digits)

Introduce the **Formal Written Method**.

1) No carrying required (TU+TU)
2) Carrying required (TU+TU)
3) Carrying required (HTU+HTU)

The digits that have been ‘carried’ should be recorded under the line in the correct column.

1) 63 + 32 =

\[
\begin{array}{c}
63 \\
+ 32 \\
\hline
95
\end{array}
\]

2) 44 + 76 =

\[
\begin{array}{c}
44 \\
+ 76 \\
\hline
120
\end{array}
\]

‘Three add two equals five. Write five in the ones/units column. 60 add 30 equals 90. Write 9 (90) in the tens column. The total is 95.’

3) 476 + 147 = 623

\[
\begin{array}{c}
447 \\
+ 176 \\
\hline
623
\end{array}
\]

‘Seven add six equals 13. Write three in the units column and ‘carry’ one across into the tens column (10). 40 add 70 and the ten that we carried equals 120. Write 2 in the tens column (20) and ‘carry’ 1 across into the hundreds column (100). 400 add 100 and the 100 that has been carried equals 600. Write 6 in the hundreds column (600). The total is 623.’

When children are confident, introduce addition of a four-digit number and a three-digit number:

1,845 + 526 = 2,371

\[
\begin{array}{c}
1845 \\
+ 526 \\
\hline
2371
\end{array}
\]
Continue to develop with addition of 2 four-digit numbers and with decimal numbers, in the context of money or measurement. Ensure that the decimal points line up.

£45·65 + £ 28·50 = £74·15

\[
\begin{array}{c}
45\cdot65 \\
+28\cdot50 \\
\hline \\
74\cdot15 \\
\end{array}
\]
Addition - Year Five & Year 6 (Revision)

Introduce the use of **empty number lines** with larger numbers and decimal numbers, as appropriate.

Continue to develop the **formal written method for addition** with larger numbers (and decimal numbers) and with the addition of three or more numbers:

\[
21,848 + 1,523 = 23,371
\]

\[
\begin{array}{c}
21848 \\
+ 1523 \\
\hline
23371
\end{array}
\]

Ensure that the digits that have been ‘carried’ are recorded under the line in the correct column.

Use the **formal written method** for the addition of decimal numbers:

\[
£154.75 + £233.82 = £388.57
\]

\[
\begin{array}{r}
154.75 \\
+ 233.82 \\
\hline
388.57
\end{array}
\]

Ensure that the decimal points line up.

Continue to practise and apply the formal written method throughout Y5, including the addition of more than two numbers.
Bar Method (Year 3-6)

Continue to build on understanding from Inspire Maths (Year 3 only) or introduce the bar method (Year 4-6).
Review understanding of part + part = whole.

For one-step problems/missing number problems, review bar model strategies for **aggregation** and **augmentation**:

For two-step/multi-operational problems, review before (step 1) and after (step 2) strategies.

Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?

A tub contains £24.00. Saj takes £5.00. Joss takes £10.00 coins. How much money is left in the tub?
Subtraction

Subtraction - Year Three

Number Line Method

Introduce **empty number line strategy**.

\[ 76 - 45 = 31 \]

Extend with larger numbers by counting back...

\[ 216 - 27 = 189 \]
**Finding a Small Difference**

Use complementary addition to find differences (only use for **small** differences). **Count up** from the smallest number to the largest to **find the difference**.

31 - 28 = 3

- 28
- 29
- 30
- 31

‘The difference between 28 and 31 is 3.’

If children are confident, further develop this method to find the difference, using more efficient jumps:

74 – 58 = 16

- 58
- 60
- 70
- 74

‘The difference between 58 and 74 is 16.’
Formal Written Method (2 digits - 2digits, 3 digits - 3 digits)

Continue to build on understanding from ‘Inspire Math’s using Formal Written Method.

1) No exchanging required (TU-TU)
2) Exchanging required (TU-TU)
3) Exchanging required (HTU-HTU)

The digits that have been ‘exchanged’ should be recorded at the top of the calculation.

1) 78 - 23 =

```
  7 8
- 2 3
  5 5
```

\[ \text{Eight subtract three is five, seventy subtract twenty is fifty. The answer is fifty-five} \]

2) 73 - 27 =

```
6  1
7  3
- 2  7
  4  6
```

\[ \text{We can’t subtract seven from three, so we need to exchange a ten for ten ones to give us 60 + 13.} \]

\[ \text{73 is partitioned into 60+13 in order to calculate 73-27.} \]

3) 235 - 127 =

```
2  1
2 3 5
- 1 2 7
  1 0 8
```

\[ \text{In this example it has only been necessary to exchange from the tens column.} \]
**Subtraction - Year Four**

**Number Line Method (3d/4d)**

Introduce *empty number line strategy* and review Year 3 method (3d or 4d).

\[6258 - 1273 = 185\]

![Number Line Diagram]

**Formal Written Method (3d/4d)**

The digits that have been ‘exchanged’ should be recorded at the top of the calculation.

\[
\begin{array}{c}
51 \\
637 \\
-252 \\
385
\end{array}
\]

When children are confident, develop further with four-digit numbers and decimal numbers, in the context of money and measures. Ensure that the decimal points line up.

\[
\begin{array}{c}
3,625 - 1,219 = 2,406 \\
\$56.75 - \$34.80 = \$21.95
\end{array}
\]

\[
\begin{array}{c}
3625 \\
-1219 \\
2406
\end{array}
\]

\[
\begin{array}{c}
51 \\
56.75 \\
-34.80 \\
21.95
\end{array}
\]
Subtraction - Year Five & Year 6 (Revision)

Introduce the use of empty number lines with larger numbers and decimal numbers, as appropriate.

**Formal Written Method (4 digits/decimals)**

Continue to develop the formal written method for subtraction with larger numbers (and decimal numbers) and with the subtraction of three or more numbers:

\[
12,731 \, - \, 1,367 = 11,364
\]

\[
\begin{array}{c@{}c@{}c@{}c@{}c}
 & 1 & 2 & 7 & 3 \\
- & 1 & 3 & 6 & 7 \\
\hline
 & 1 & 1 & 3 & 6 & 4
\end{array}
\]

\[
£166.25 \, - \, £83.72 = £82.53
\]

\[
\begin{array}{c@{}c@{}c@{}c@{}c}
1 & 6 & 6 & 2 & 5 \\
- & 8 & 3 & 7 & 2 \\
\hline
8 & 2 & 5 & 3
\end{array}
\]
Bar Method (Year 3-6)

Continue to build on understanding from Inspire Maths (Year 3 only) or introduce the bar method (Year 4-6). Review understanding of part + part = whole.

For one-step problems/missing number problems, review bar model strategies for Take Away and Comparison/Difference:

There are 561 cars in a supermarket car park. 267 cars leave. How many cars are left?
Multiplication

Multiplication - Year Three

Arrays
Continue to build on understanding from ‘Inspire Math’s arrays to support multiplication, as appropriate.

Number Line Method
Introduce empty number line strategy to count on:

4 x 3 = 12
‘0, 3, 6, 9, 12’

1x3  2x3  3x3  4x3

0  3  6  9  12
**Partitioning**

Introduce partitioning to support grid method (multiplication) and chunking (division).

13 x 5 = 65 (Partition 13 into 10 + 3)

10 x 5 = 50
3 x 5 = 15

50 + 15 = 65

**Grid Method (2dx1d)**

Support children with construction of the grid.

13 x 8 = 104

‘Partition 13 into 10 + 3 then multiply each number by 8. Add the partial products (80 and 24) together.’

<table>
<thead>
<tr>
<th>X</th>
<th>10</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>80</td>
<td>24</td>
</tr>
</tbody>
</table>

80 + 24 = 104

Children to use mental addition to re-combine the partial products.

36 x 4 = 144

<table>
<thead>
<tr>
<th>X</th>
<th>30</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>120</td>
<td>24</td>
</tr>
</tbody>
</table>

120 + 24 = 144 (add the partial products)
**Expanded Short Multiplication (2dx1d)**

Introduce **expanded short multiplication** to support formal short method of multiplication and chunking (division).

\[
13 \times 8 = 104
\]

\[
\begin{array}{c}
10 + 3 \\
\times 8 \\
\hline
24 \quad (3 \times 8) \\
+ 80 \quad (10 \times 8) \\
\hline
104
\end{array}
\]

**Formal Short Multiplication (2dx1d)**

\[
\begin{array}{c}
13 \\
\times 8 \\
\hline
104
\end{array}
\]

Ensure that the digit ‘carried over’ is written under the line in the correct column.
Multiplication - Year Four

If necessary, return to the grid method and/or expanded short or formal method first. Introduce additional ‘hundreds' column to each strategy.

**Grid Method (3dx1d)**

If necessary, return to the grid method and/or expanded method first:

\[ 127 \times 6 = 762 \]

<table>
<thead>
<tr>
<th>( \times )</th>
<th>100</th>
<th>20</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>600</td>
<td>120</td>
<td>42</td>
</tr>
</tbody>
</table>

600 + 120 + 42 = 762 (add the partial products)

600
120
+ 42
762

Children to use mental addition to recombine the partial products OR column addition to add partial products.

**Expanded Short Multiplication (3dx1d)**

\[ 127 \times 6 = 762 \]

1 2 7
\times 6
4 2 (6 x 7)
+ 1 2 0 (6 x 20)
+ 6 0 0 (6 x 100)

7 6 2
Formal Short Multiplication (3x1d)

1 2 7
x  6
7 6 2
1 4
Multiplication - Year Five

If necessary, return to the grid method and/or expanded short or formal method first. Introduce additional ‘thousands’ column to each strategy – and the additional row for the 2dx2d grid method.

**Formal Short Multiplication (4dx1d)**

\[ 1,256 \times 4 = 5,024 \]

```
  1 2 5 6
x     4
---
  5 0 2 4
  1 2 2
```

**Grid Method (2dx2d)**

\[ 23 \times 13 = (20 + 3) \times (10 + 3) = 299 \]

<table>
<thead>
<tr>
<th></th>
<th>20</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>200</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>9</td>
</tr>
</tbody>
</table>

\[ = 2 3 0 \]
\[ + 6 9 \]
\[ = 2 9 9 \]

Add the partial products \( (200 + 30) + (60 + 9) = 299 \)
**Expanded Long Multiplication (2dx2d)**

Introduce **expanded long multiplication** to support formal short method of multiplication and chunking (division).

\[
23 \times 13 = 299
\]

\[
\begin{array}{c}
23 \\
\times 13 \\
\hline
9 \quad (3 \times 3) \\
60 \quad (3 \times 20) \\
+30 \quad (10 \times 3) \\
\hline
200 \quad (10 \times 20) \\
299
\end{array}
\]

**Compact Long Multiplication (2dx2d)**

\[
56 \times 27 = 1,512
\]

\[
\begin{array}{c}
56 \\
\times 27 \\
\hline
3942 \quad (7 \times 56) \\
+11120 \quad (20 \times 56) \\
\hline
1512 \\
1
\end{array}
\]

In this example there are digits that have been ‘carried’ over in the partial products.

The prompts (in brackets) can be omitted if children no longer need them.

**Short Multiplication (2dx2d)**

\[
3,256 \times 4 = 13,024
\]

\[
\begin{array}{c}
3256 \\
\times 4 \\
\hline
13024 \\
122
\end{array}
\]
Multiplication - Year Six (Revision)

Short Multiplication (2dx2d)

3,256 x 4 = 13,024

```
  3 2 5 6
x   4
  1 3 0 2 4
  1 2 2
```

Expanded Long Multiplication (decimals)

Introduce the formal written method of long multiplication using decimals.

53\cdot2 \times 24 = 1,276\cdot8

```
  5 3 \cdot 2
x   2 4
  2 1 2 \cdot 8 \quad (53\cdot2 \times 4)
 1 0 6 4 \cdot 0 \quad (53\cdot2 \times 20)
 1 2 7 6 \cdot 8
```

The prompts (in brackets) can be omitted if children no longer need them.
Bar Method (Year 3-6)

Continue to build on understanding from Inspire Maths (Year 3 only) or introduce the bar method (Year 4-6).

Groups/Sets/Lots/Repeated Addition (Groups/Sets/Lots)

8 children each download 59 songs to play on their iPod. How many songs do they have altogether?

Integer Scaling

Peter has 4 books
Harry has five times as many books as Peter. How many books has Harry?

4

4 4 4 4 4

4 x 5 = 20
Harry has 20 books
Division

Division - Year Three

Arrays
Continue to use arrays to support multiplication, as appropriate.

\[
\begin{align*}
\begin{array}{cccccccc}
\text{Array} & \text{Array} & \text{Array} & \text{Array} & \text{Array} & \text{Array} & \text{Array} & \text{Array} \\
\begin{array}{cccc}
\text{Array} & \text{Array} & \text{Array} & \text{Array} \\
\text{Array} & \text{Array} & \text{Array} & \text{Array} \\
\text{Array} & \text{Array} & \text{Array} & \text{Array} \\
\text{Array} & \text{Array} & \text{Array} & \text{Array} \\
\text{Array} & \text{Array} & \text{Array} & \text{Array} \\
\text{Array} & \text{Array} & \text{Array} & \text{Array} \\
\text{Array} & \text{Array} & \text{Array} & \text{Array} \\
\end{array}
\end{array}
\end{align*}
\]

\[
\begin{align*}
20 \div 5 &= 4 \\
20 \div 4 &= 5
\end{align*}
\]

Number Line Method

Introduce empty number line strategy to jump back to make the link with repeated subtraction

\[
24 \div 3 = 8
\]

\[
\begin{array}{cccccccc}
0 & 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24
\end{array}
\]

Introduce empty number line strategy to count forwards…

‘How many threes are there in 24?’
Begin to determine **remainders**:

\[ 25 \div 3 = 8 \text{ r} 1 \]

Eight jumps of three and one left over.

Alternatively, you could jump forwards in multiples of three from zero to twenty-four (‘and one more makes 25’)

**Formal Written Method (Bus Stop/Division Bar)**

\[ 24 \div 3 = 8 \]

\[
\begin{array}{c}
24 \\
\hline
3 \overline{)24}
\end{array}
\]

‘Twenty four divided by three equals eight.’
‘How many threes are there in twenty four?’

\[ 33 \div 8 = 4 \text{ r} 1 \]

\[
\begin{array}{c}
33 \\
\hline
8 \overline{)33}
\end{array}
\]

This could be modelled using an empty number line to ensure understanding, if needed (See Y3 guidance)
Division - Year Four

Partitioning (2d ÷ 1d)

65 ÷ 5 = 13

65 = 50 + 15  Partition 65 into 50 and 15

50 ÷ 5 = 10
15 ÷ 5 = 3
10 + 3 = 13

Partitioning using formal layout (2d ÷ 1d)

98 ÷ 7 = 14

'\(90 = 70 + 28\).
10 + 4 = 14
7 \(70 + 28\)

Seven ‘goes into’ 70 ten times and seven ‘goes into’ 28 four times. Ten add four equals 14'
**Chunking using formal layout (2d ÷ 1d)**

144 ÷ 16 = 9

\[
\begin{array}{c}
\phantom{-}144 \\
\underline{16} \\
\phantom{-}64 \\
- 64 \quad (4 \times 16) \\
\phantom{-}80 \\
\underline{80} \\
\phantom{-}0 \\
\end{array}
\]

- Multiples of the divisor (16) have been subtracted from the dividend (144)
- ‘4 (lots of 16) + 4 (lots of 16) + 1 (lot of 16) = 9 (lots of 16).’
- The answer is nine.
- There is no remainder

**Formal Written Method of Short Division**

98 ÷ 7 = 14

\[
\begin{array}{c}
\phantom{-}98 \\
\underline{7} \\
\phantom{-}28 \\
\end{array}
\]

- Use the vocabulary of place value to ensure understanding and make the link to partitioning.

132 ÷ 6 = 22

\[
\begin{array}{c}
\phantom{-}132 \\
\underline{6} \\
\phantom{-}12 \\
\end{array}
\]

- Use the language of place value to ensure understanding.
Division - Year Five

Formal Written Method of Short Division (with remainders)

184 ÷ 8 = 23

\[ 8 \overline{)184} \]
\[
\begin{array}{r}
23 \\
\hline
18 \\
4
\end{array}
\]

Use the language of place value to ensure understanding.

Make the link to the partitioning method (see Y4 guidance).

432 ÷ 5 = 86 r2

\[ 5 \overline{)432} \]
\[
\begin{array}{r}
86 \text{ r } 2 \\
\hline
432
\end{array}
\]

The remainder can also be expressed as a fraction, 2/5 (the remainder divided by the divisor): 432 ÷ 5 = 86 2/5 or 86.4
Division - Year Six

Formal Written Method of Short Division (with remainders) (3d÷2d)

\[ 496 \div 11 = 45 \text{ r} 1 \]

\[
\begin{array}{c}
\text{11) } 4 9 6 \\
\text{- 4 4 0 (40 x 11)} \\
\text{5 6} \\
\text{- 5 5 (5 x 11)} \\
\text{1 (remainder)}
\end{array}
\]

Formal Written Method of Long Division (with remainders) (3d÷2d)

\[
\begin{array}{c}
\text{11) } 4 5 \text{ r} 1 \\
\text{4 9 6} \\
\text{- 4 0 (lots of 11)} \\
\text{5 6} \\
\text{- 5 5 (lots of 11)} \\
\text{1 (remainder)}
\end{array}
\]

Multiples of the divisor (11) have been subtracted from the dividend (496)

'40 (lots of 11) + 5 (lots of 11) = 45 (lots of 11)'

'1 is the remainder'

Long division using decimal remainders

\[ 432 \div 15 = 28 \cdot 8 \]

\[
\begin{array}{c}
\text{15) } 4 3 2 \cdot 0 \\
\text{3 0} \\
\text{1 3 2} \\
\text{1 2 0} \\
\text{1 2 0} \\
\text{0}
\end{array}
\]

Only teach this method when children are completely secure with the previous method.

The remainder is expressed as a decimal.
Alternative/Revision Chunking using formal layout (2d ÷ 2d)

144 ÷ 16 = 9

```
16 | 1 4 4  
   - 6 4   (4 × 16)  
     8 0  
   - 6 4   (4 × 16)  
     1 6  
   - 1 6   (1 × 16)  
     0  
```

Multiples of the divisor (16) have been subtracted from the dividend (144)

'4 (lots of 16) + 4 (lots of 16) + 1 (lot of 16) = 9 (lots of 16).

The answer is nine.

There is no remainder

Chunking using formal layout (3d ÷ 2d with remainders)

432 ÷ 15 = 28 r12

```
15 | 4 3 2  
   - 3 0 0   (20 × 15)  
     1 3 2  
   - 1 2 0   (8 × 15)  
     1 2   (remainder)  
```

Multiples of the divisor (15) have been subtracted from the dividend (432)

'20 (lots of 15) + 8 (lots of 15) = 28
12 is the remainder'

The remainder can also be expressed as a fraction, 12/15 which can be simplified to 4/5, or as a decimal, 0.8 (See next example)
Bar Method (Year 3-6)

Fractions

e.g. A computer game is £24 in the sale. This is one quarter of its original price. How much did it

Ratio

e.g. At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many

Percentages

e.g. There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?